Economic and Regulatory Capital in Banking: What Is the Difference?*

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We analyze the determinants of regulatory capital (the minimum required by regulation), economic capital (that chosen by shareholders without regulation), and actual capital (that chosen with regulation) in a dynamic model of a bank with a loan-portfolio return described by the single-risk-factor model of Basel II. We show that variables that only affect economic capital, such as the intermediation margin and the cost of capital, can account for large deviations from regulatory capital. Actual capital is closer to regulatory capital, but the threat of closing undercapitalized banks generates significant capital buffers. Market discipline, proxied by the coverage of deposit insurance, increases economic and actual capital, although the effects are small.

JEL Codes: G21, G28.

1. Introduction

Economic capital and *regulatory capital* are two terms frequently used in the analysis of the new framework for bank capital regulation proposed by the Basel Committee on Banking Supervision (2004). Known as Basel II, this framework is in the process of being implemented worldwide. According to the chairman of the Basel Committee, the primary objective of the new regulation is to set

*We thank Jaime Caruana, Douglas Gale, Charles Goodhart, Esa Jokivuolle, Nobu Kiyotaki, Rosa Lastra, Julio Segura, Andrew Winton, and especially Javier Suarez for helpful comments. This paper was written when the first author was a PhD student at CEMFI and Universidad Pública de Navarra. The views expressed in the paper are those of the authors. Address: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. E-mail: abel_elizalde@hotmail.com, repullo@cemfi.es.

"... more risk-sensitive minimum capital requirements, so that regulatory capital is both adequate and closer to economic capital" (Caruana 2005, 9).

To compare economic and regulatory capital, we must first clarify the meaning of each term. In principle, regulatory capital should be derived from the maximization of a social welfare function that takes into account the costs (e.g., an increase in the cost of credit) and the benefits (e.g., a reduction in the probability of bank failure) of capital regulation.[1] However, in this paper we define *regulatory capital* as the minimum capital required by the regulator, which we identify with the capital charges in the internal-ratings-based (IRB) approach of Basel II. Economic capital is usually defined as the capital level that is required to cover the bank's losses with a certain probability or confidence level, which is related to a desired rating.[2] However, it is our view that such desired solvency standard should not be taken as a primitive, but should be derived from an underlying objective function such as the maximization of the value of the bank. For this reason, *economic capital* may be defined as the capital level that bank shareholders would choose in absence of capital regulation.[3] This is, in fact, the definition we will use hereafter.

The purpose of this paper is to analyze the differences between economic and regulatory capital in the context of a dynamic model of a bank with a loan-portfolio return described by the single-risk-factor model that underlies the IRB capital requirements of Basel II. Economic capital is the level of capital chosen by shareholders at the beginning of each period in order to maximize the value of the bank, taking into account the possibility that the bank will be closed if the losses during the period exceed the initial level of capital. This closure rule may be justified by assuming that a bank run takes place before the shareholders can raise new equity to cover the losses. Thus economic capital trades off the costs of funding the

[1]See Repullo and Suarez (2004) for a discussion of Basel II from this perspective.

[2]See, e.g., Jones and Mingo (1998) or Carey (2001).

[3]As noted by Allen (2006, 45), "The two concepts reflect the needs of different primary stakeholders. For economic capital, the primary stakeholders are the bank's shareholders, and the objective is the maximization of [their] wealth. For regulatory capital, the primary stakeholders are the bank's [depositors], and the objective is to minimize the possibility of loss."

bank with costly equity against the benefits of reducing the probability of losing the bank's franchise value, which appears as a key endogenous variable in the bank's maximization problem.

We show that economic and regulatory capital do not depend on the same variables: the former (but not the latter) depends on the intermediation margin and the cost of bank capital, while the latter (but not the former) depends on the confidence level set by the regulator. Moreover, economic and regulatory capital do not respond in the same manner to changes in the common variables that affect them, such as the loans' probability of default and loss given default.

Due to the difficulty of obtaining analytical results for economic capital, we use a numerical procedure to compute it. The results show that Basel II regulatory capital only approaches economic capital for a limited range of parameter values. Our analysis also shows that the relative position of economic and regulatory capital is mainly determined by the cost of bank capital: economic capital is higher (lower) than regulatory capital when the cost of capital is low (high).

Another key variable in the shareholders' economic capital decision is the intermediation margin, which has two opposite effects. On the one hand, a higher margin increases the bank's franchise value and, consequently, shareholders' incentives to contribute capital. On the other hand, a higher margin increases bank revenues and therefore reduces the role of capital as a buffer to absorb future losses, acting as a substitute for economic capital. We show that the net effect of the intermediation margin on economic capital is positive in very competitive loan markets and negative otherwise. Finally, the numerical results show that increases in the loans' probability of default and loss given default increase regulatory capital, while they only increase economic capital for a range of plausible values of these variables.

The paper also addresses the determinants of *actual capital*, which is defined as the capital chosen by bank shareholders taking into account regulatory constraints. In particular, two regulations are considered. First, at the beginning of each period, banks must have a capital level no lower than regulatory capital. Second, following U.S. banking regulation and in particular the prompt corrective action (PCA) provisions of the Federal Deposit Insurance Corporation Improvement Act (FDICIA), banks whose capital level at the end of a period falls below a minimum (positive) threshold are considered critically undercapitalized and are closed. We show that, for a wide range of parameter values, the threat of closing undercapitalized banks induces them to choose a capital level above regulatory capital. Therefore, in situations in which the cost of capital is such that economic capital is below regulatory capital, PCA provides an explanation for why banks typically hold a buffer of capital above the minimum required by regulation.[4]

The model proposed in the paper allows us to analyze the effect of market discipline, proxied by the coverage of deposit insurance, on economic and actual capital. We consider two alternative scenarios: one in which depositors are fully insured and where the deposit interest rate is equal to the risk-free rate, and another one in which depositors are uninsured. In this second scenario, depositors require an interest rate such that the expected return of their investment is equal to the risk-free rate. The results suggest that measures aimed at increasing market discipline have a positive effect on economic capital, though the magnitude of this effect is generally small, except in very competitive markets for high-risk loans. The impact of market discipline on actual capital is even lower and almost negligible.

Two limitations of our analysis are the assumption that the risk of the bank's loan portfolio is exogenous and the use of the single-risk-factor model to derive the probability distribution of loan default rates. The inclusion of the bank's level of risk as an endogenous variable, together with capital, in the shareholders' maximization problem, as well as the analysis of more-complex models of bank risk are left for future research.

The academic literature on this topic is small, and in no case economic and regulatory capital are compared. The literature, both empirical and theoretical, deals with the impact of different regulations on capital (as we do) and risk-taking decisions (as we do not do).

[4] An alternative explanation would be the banks' incentives to maintain high credit ratings to support their derivatives counterparty business; see Jackson, Perraudin, and Saporta (2002). This could be modeled as an extra revenue that comes from this business as long as the probability of failure is sufficiently low (or the rating is sufficiently high). We plan to explore this issue in future work.

From an empirical perspective, it is not possible to analyze economic capital in the sense defined above, given that some form of capital regulation has been in place for many years. In terms of actual capital, the predictions of our model coincide with several stylized facts supported by empirical studies of the drivers of banks' capital. Flannery and Rangan (2004) analyze the relationship between regulatory and actual bank capital between 1986 and 2000 for a sample of U.S. banks. They conclude that the increase in regulatory capital during the first part of the 1990s could explain the increase in the capital levels of the banking industry during those years, but that the additional increase in capital in the second part of the 1990s is mainly driven by market discipline. These results support two predictions of our model: actual capital is an increasing function of regulatory capital and of the level of market discipline. However, our results suggest that mandatory restrictions and penalties to undercapitalized banks could have played a more important role in boosting bank capital levels than market discipline. This is in line with the evidence provided by Aggarwal and Jacques (2001) showing that both adequately capitalized and undercapitalized banks increased their capital ratios in response to the PCA provisions of FDICIA during both the announcement period, 1992, and the years after the standards went into effect, 1993–96.

From a theoretical perspective, the literature provides a wide range of assumptions and modeling frameworks, which complicates the comparison with ours (and between them). There are a few papers that share the focus on dynamic models with endogenous franchise values. The most interesting paper—which is the foundation of our analysis of economic capital—is Suarez (1994), who constructs a dynamic model of bank behavior in which shareholders choose not only the capital level but also the asset risk.

Calem and Rob (1999) present a dynamic model similar to ours where the bank's franchise value is endogenous. The model is calibrated with empirical data from the banking industry for 1984–93, focusing on the impact of risk-based versus flat-rate capital requirements on banks' risk taking, which is shown to be ambiguous across banks depending on their capital levels. Repullo (2004) analyzes capital and risk-taking decisions in a dynamic model of imperfect competition with endogenous franchise values. He shows that capital requirements reduce the banks' incentives to take risk, and that risk-based requirements are more-efficient regulatory tools. Estrella (2004) presents a dynamic model in which banks choose their capital subject to risk-based capital regulation and adjustment costs in both raising capital and paying dividends. He focuses on capital levels over the cycle, concluding that risk-based capital regulation, if binding, is likely to be procyclical.

This paper is organized as follows. Section 2 presents the model and characterizes the determinants of economic, regulatory, and actual capital. Section 3 derives the numerical results, and section 4 concludes. Appendix 1 discusses the comparative statics of economic capital, and appendix 2 contains a proof of the negative relationship between bank capital and the interest rate on uninsured deposits.

2. The Model

Consider a bank that, at the beginning of each period $t = 0, 1, 2, \ldots$ in which it is open, has an asset size that is normalized to 1.[5] The bank is funded with deposits, $1 - k_t$, that promise an interest rate c, and capital, k_t, that requires an expected return δ. We assume that the deposit rate c is smaller than the cost of capital δ. The bank is owned by risk-neutral shareholders who have limited liability and, in the absence of minimum capital regulation, choose the capital level k_t in the interval $[0, 1]$. When $k_t = 0$, the bank is fully funded with deposits, and when $k_t = 1$, the bank is fully funded with equity capital.

In each period t in which the bank is open, its funds are invested in a portfolio of loans paying an exogenously fixed interest rate r. The return of this investment is stochastic: a random fraction $x_t \in [0, 1]$ of these loans default, in which case the bank loses the interest r as well as a fraction $\lambda \in [0, 1]$ of the principal. Therefore, the bank gets $1 + r$ from the fraction $1 - x_t$ of the loans that do not default, and it recovers $1 - \lambda$ from the fraction x_t of defaulted loans, so the value of its portfolio at the end of period t is given by

$$a_t = (1 - x_t)(1 + r) + x_t(1 - \lambda). \tag{1}$$

[5]This normalization is related to the size of the bank's loan customer base, which is assumed to be fixed over time. Introducing a (small) growth rate of the customer base would be straightforward and would not change the qualitative results.

Since the bank has to pay depositors an amount $(1-k_t)(1+c)$, assuming zero intermediation costs, its capital at the end of period t is

$$k_t' = a_t - (1-k_t)(1+c) = k_t + r - (1-k_t)c - (\lambda + r)x_t. \qquad (2)$$

We assume that the probability distribution of the default rate x_t is the one derived from the single-risk-factor model of Vasicek (2002) that is used for the computation of the capital charges in the IRB approach of Basel II.[6] Its cumulative distribution function is given by

$$F(x_t) = N\left(\frac{\sqrt{1-\rho}\; N^{-1}(x_t) - N^{-1}(p)}{\sqrt{\rho}}\right), \qquad (3)$$

where $N(\cdot)$ denotes the distribution function of a standard normal random variable, $p \in [0,1]$ is the loans' (unconditional) probability of default, and $\rho \in [0,1]$ is their exposure to the systematic risk factor: when $\rho = 0$, defaults are statistically independent, so $x_t = p$ with probability 1, and when $\rho = 1$, defaults are perfectly correlated, so $x_t = 0$ with probability $1-p$, and $x_t = 1$ with probability p. We also assume that the default rate x_t is independent over time.

The distribution function $F(x_t)$ in (3) is increasing, with $F(0) = 0$ and $F(1) = 1$. Moreover, it can be shown that

$$E(x_t) = \int_0^1 x_t \; dF(x_t) = p$$

and

$$Var(x_t) = \int_0^1 (x_t - p)^2 \; dF(x_t) = N_2(N^{-1}(p), N^{-1}(p); \rho) - p^2,$$

where $N_2(\cdot,\cdot;\rho)$ denotes the distribution function of a zero-mean bivariate normal random variable with standard deviation equal to 1 and correlation coefficient ρ; see Vasicek (2002). Therefore, the

[6] As shown by Gordy (2003), this model has the property that the contribution of a given asset to value-at-risk (and hence the corresponding IRB capital charge) is portfolio invariant; i.e., it depends on the asset's own characteristics and not on those of the portfolio in which it is included.

expected value of the default rate is the probability of default p, while its variance is increasing in the correlation parameter ρ, with $Var(x_t) = 0$ for $\rho = 0$, and $Var(x_t) = p(1-p)$ for $\rho = 1$.

2.1 Economic Capital

To derive the level of capital chosen by the bank shareholders in the absence of minimum capital regulation, we solve a dynamic programming problem in which the state variable $I_t \in \{0,1\}$ indicates whether the bank is closed ($I_t = 0$) or open ($I_t = 1$) at the beginning of period t. Let $V(I_t)$ denote the value function of this problem. Clearly, the value $V(0)$ of a bank that is closed is 0, while $V(1)$ is the franchise value of a bank that is open, which henceforth will simply be denoted by V.

Following Suarez (1994), a closure rule may be described by a function $h\colon \mathbb{R} \to \{0,1\}$ that specifies the values of end-of-period capital k'_t for which a bank that is open at the beginning of period t is closed at the end of this period, i.e.,

$$I_{t+1} = I_t h\left(k'_t\right). \tag{4}$$

Notice that $I_t = 0$ implies $I_{t+1} = 0$, so a bank that is closed cannot be reopened.

Two particular closure rules will be considered. The first rule assumes that the shareholders can freely recapitalize the bank when its capital at the end of period t is negative, so the bank is only closed when its shareholders do not exercise the recapitalization option. This will happen whenever the funds that they have to inject to pay back depositors are greater than the franchise value of the bank, i.e., whenever $k'_t + V < 0$. The second closure rule assumes that the bank is closed at the end of period t whenever $k'_t < 0$, i.e., whenever the losses during the period exceed the initial capital k_t. The rationale for this rule is that when the liabilities take the form of demandable deposits, a shock that depletes all the bank's capital triggers a run before the shareholders are able to raise fresh equity to cover the shortfall.

Formally, the first closure rule is described by

$$h_1\left(k'_t\right) = \begin{cases} 0 & \text{if } k'_t + V < 0, \\ 1 & \text{otherwise,} \end{cases} \tag{5}$$

while the second is described by

$$h_2(k_t') = \begin{cases} 0 & \text{if } k_t' < 0, \\ 1 & \text{otherwise.} \end{cases} \tag{6}$$

The Bellman equation that characterizes the solution to the bank's maximization problem for closure rule (5) is

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} E\big(\max \left\{ k_t' + V, 0 \right\} \big) \right]. \tag{7}$$

According to this expression, the franchise value V of a bank that is open results from maximizing with respect to k_t an objective function that has two terms: the first one, with a negative sign, is the capital contribution of the shareholders at the beginning of period t; the second one is the discounted expected payoff at the end of period t, which comprises the value k_t' of its end-of-period capital plus the value V of remaining open in period $t+1$, whenever their sum $k_t' + V$ is non-negative, and 0 otherwise. Notice that the discount rate used in this second term is the return required by bank shareholders, or cost of capital δ.

Therefore, assuming that the bank is open at the beginning of period t, there are two possible scenarios at the end of period t: (i) if $k_t' + V < 0$, the bank fails and the shareholders get a final payoff of 0, and (ii) if $k_t' + V \geq 0$, the bank remains open in period $t+1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k_t' - k_{t+1}$, i.e., the difference between the capital at the end of period t and the capital that they would like to keep in the bank for period $t+1$.

Using the definition of k_t', (2), we have $k_t' + V \geq 0$ if and only if the default rate x_t is below the critical value

$$x(k_t, V) = \frac{k_t + r - (1-k_t)c + V}{\lambda + r}, \tag{8}$$

so we can rewrite Bellman equation (7) as

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \int_0^{x(k_t,V)} \times \left[k_t + r - (1-k_t)c - (\lambda + r)x_t + V \right] \, dF(x_t) \right]. \tag{9}$$

Differentiating the bank's objective function with respect to k_t, and using the assumption that the deposit rate c is smaller than the cost of capital δ, we get

$$-1 + \frac{1}{1+\delta} \int_0^{x(k_t,V)} (1+c)\, dF(x_t) \leq -1 + \frac{1+c}{1+\delta} = \frac{c-\delta}{1+\delta} < 0.$$

Hence we conclude that for the first closure rule, (5), shareholders always choose the corner solution $k_t = 0$, i.e., zero economic capital. The intuition for this result is clear: bank shareholders do not have an incentive to contribute costly capital ex ante when they are able to provide it ex post via the recapitalization option.

However, as argued above, raising equity capital after a large negative shock may not be feasible, especially when the liabilities take the form of demandable deposits that may be subject to runs. For this reason, in what follows we focus exclusively on the second closure rule, (6), according to which the bank is closed at the end of period t whenever $k_t' < 0$.

The Bellman equation that characterizes the solution to the bank's maximization problem for closure rule (6) is

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \left[E\big(\max\left\{ k_t', 0 \right\}\big) + \Pr(k_t' \geq 0) V \right] \right]. \quad (10)$$

According to this expression, the franchise value V of a bank that is open results from maximizing with respect to k_t an objective function that has three terms: the first one, with a negative sign, is the capital contribution of the shareholders at the beginning of period t; the second one is the discounted expected payoff at the end of period t; and the third one is the discounted expected value of remaining open in period $t+1$. As before, the discount rate used in the last two terms is the return required by bank shareholders, or cost of capital δ.

Therefore, assuming that the bank is open at the beginning of period t, there are two possible scenarios at the end of period t: (i) if $k_t' < 0$, the bank fails and the shareholders get a final payoff of 0, and (ii) if $k_t' \geq 0$, the bank remains open in period $t+1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k_t' - k_{t+1}$, i.e., the difference between the capital at the end of period t and the capital that they would like to keep in the bank for period $t+1$.

Using the definition of k_t', (2), we have $k_t' \geq 0$ if and only if the default rate x_t is below the critical value

$$x(k_t) = \frac{k_t + r - (1-k_t)c}{\lambda + r}, \tag{11}$$

so we can rewrite Bellman equation (10) as

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \int_0^{x(k_t)} \right.$$
$$\left. \times \; [k_t + r - (1-k_t)c - (\lambda + r)x_t + V] \; dF(x_t) \right]. \tag{12}$$

Notice that for

$$k_t \geq k_{\max} = \frac{\lambda + c}{1 + c}, \tag{13}$$

we have $x(k_t) \geq 1$, so the probability of bank failure is 0. In this case the derivative with respect to k_t of the bank's objective function equals $(c-\delta)/(1+\delta)$, which is negative by the assumption that the deposit rate c is smaller than the cost of capital δ. Hence economic capital will never be above $k_{\max}$. This result is easy to explain: bank shareholders might be willing to contribute capital, instead of funding the bank with cheaper deposits, as long as capital provides a buffer that reduces the probability of failure and consequently increases the probability of receiving a stream of future dividends. However, if $k_t \geq k_{\max}$, capital covers the bank's losses at the end of period t even when 100 percent of the loans in its portfolio default, which means that any additional capital will only increase the bank's funding costs without reducing its probability of failure (which is 0).

The solution of Bellman equation (12) gives the level of economic capital k^* that bank shareholders would like to hold in the absence of minimum capital regulation, as well as the bank's franchise value V^*. In addition, this equation allows us to identify the determinants of economic capital k^*, which are the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ; the loan rate r; the deposit rate c; and the cost of bank capital δ.

Appendix 1 shows that economic capital can be at the corner $k^* = 0$ and that if there is an interior solution, comparative static results are, in general, ambiguous, except for the cost of capital δ, for which we obtain

$$\frac{\partial k^*}{\partial \delta} < 0.$$

Thus the higher the bank's equity funding costs, the lower the capital provided by its shareholders.

2.2 Regulatory Capital

As noted above, in this paper we follow the IRB approach of Basel II, according to which regulatory capital must cover losses due to loan defaults with a given probability (or confidence level) $\alpha = 99.9$ percent. Specifically, let $\hat{x}$ denote the α-quantile of the distribution of the default rate x_t, i.e., the critical value such that

$$\Pr(x_t \leq \hat{x}) = F(\hat{x}) = \alpha.$$

Hence we have $\hat{x} = F^{-1}(\alpha)$, so making use of (3), we get the capital requirement

$$\hat{k} = \lambda\hat{x} = \lambda N\left(\frac{N^{-1}(p) + \sqrt{\rho}\,N^{-1}(\alpha)}{\sqrt{1-\rho}}\right). \tag{14}$$

This is the formula that appears in Basel Committee on Banking Supervision (2004, paragraph 272), except for the fact that we are assuming a one-year maturity (which implies a maturity adjustment factor equal to 1) and that the correlation parameter ρ is, in Basel II, a decreasing function of the probability of default p. It should also be noted that in the IRB approach, expected losses, λp, are to be covered with general loan-loss provisions, while the remaining charge, $\lambda(\hat{x} - p)$, should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, and thus the distinction between the expected and unexpected components of loan losses is immaterial.

From IRB formula (14), we can immediately identify the determinants of regulatory capital $\hat{k}$, which are the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ, as well as the confidence level α set by the regulator.

To analyze the effects on regulatory capital $\hat{k}$ of changes in its determinants, we differentiate function (14), which gives

$$\frac{\partial \hat{k}}{\partial p} > 0, \quad \frac{\partial \hat{k}}{\partial \lambda} > 0, \quad \text{and} \quad \frac{\partial \hat{k}}{\partial \alpha} > 0.$$

Moreover, we also get

$$\frac{\partial \hat{k}}{\partial \rho} > 0 \quad \text{if and only if} \quad N^{-1}(\alpha) + \sqrt{\rho}\, N^{-1}(p) > 0,$$

which for $\alpha = 99.9$ percent and $\rho \leq 0.24$ (the maximum value in Basel II for corporate, sovereign, and bank exposures) holds for all $p \geq 0.03$ percent (the minimum value in Basel II). Therefore, we conclude that regulatory capital $\hat{k}$ is an increasing function of its four determinants.[7]

It is important to highlight the different determinants of economic and regulatory capital. Both economic and regulatory capital depend on the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ. However, while an increase in any of these variables increases regulatory capital, its effect on economic capital is, in general, ambiguous. Moreover, economic capital depends on the loan rate r, the deposit rate c, and the cost of bank capital δ, whereas regulatory capital depends on the confidence level α set by the regulator.

2.3 *Actual Capital*

We next derive the level of capital chosen by the bank shareholders when their choice is restricted by two regulatory constraints. First, we assume that there is a supervisor that audits the bank at the beginning of each period and requires it to hold at least the regulatory capital $\hat{k}$ in order to operate. Second, in line with U.S.

[7]In contrast, regulatory capital in the 1988 Accord (Basel I) was largely independent of risk. Basel I required a minimum capital equal to 8 percent of the bank's risk-weighted assets. Two basic criteria were used to compute these weights: the institutional nature of the borrower and the collateral provided. In particular, the weights were 0 percent for sovereign risks with OECD countries, 20 percent for interbank risks, 50 percent for mortgages, and 100 percent for all other risks. See Basel Committee on Banking Supervision (1988).

regulation, and in particular the PCA provisions of FDICIA, we assume that banks whose capital at the end of a period falls below a certain critical level $\hat{k}_{\min}$ are closed by the supervisor.[8]

In this setup the Bellman equation that characterizes the solution to the shareholders' maximization problem is

$$V = \max\left\{ \max_{k_t \in [\hat{k},1]} \left[-k_t + \frac{1}{1+\delta}\Big[E\big(\max\{k_t',0\}\big) + \Pr\big(k_t' \geq \hat{k}_{\min}\big)V\Big]\right], 0\right\}. \tag{15}$$

There are two differences between this equation and the one for economic capital, (10). First, the bank is not allowed to operate with an initial capital k_t below the minimum required by regulation $\hat{k}$, so the choice of k_t is restricted to the interval $[\hat{k}, 1]$. But with this constraint the shareholders may find it optimal not to operate the bank, in which case $V = \max\{\cdot, 0\} = 0$. Second, equation (15) takes into account that the bank is closed by the supervisor when its end-of-period capital k_t' falls below the critical level $\hat{k}_{\min}$, so the discounted value of remaining open in period $t+1$ is multiplied by $\Pr(k_t' \geq \hat{k}_{\min})$.

The solution of Bellman equation (15) gives the actual level of capital k^a that bank shareholders would like to hold given the assumed regulation, as well as the corresponding franchise value V^a. This equation also identifies the determinants of actual capital k^a, which are the same six variables that determine economic capital plus the minimum capital requirement $\hat{k}$ and the critical level $\hat{k}_{\min}$.

As in the case of economic capital, actual capital can be at the corner $k^a = \hat{k}$, in which case none of the other variables matter for actual capital. And if there is an interior solution, comparative static

[8]According to FDICIA, banks whose tangible equity ratio falls below 2 percent are considered critically undercapitalized and are required to be placed in receivership or conservatorship within ninety days of becoming critically undercapitalized; see Comptroller of the Currency (1993). Tangible equity ratio is defined as tier 1 capital plus cumulative preferred stock and related surplus, less intangibles except qualifying purchased mortgage servicing rights (PMSRs), divided by total assets, less intangibles except qualifying PMSRs.

results are, in general, ambiguous, except for the cost of capital δ and the minimum capital requirement $\hat{k}$, for which we obtain

$$\frac{\partial k^a}{\partial \delta} < 0 \quad \text{and} \quad \frac{\partial k^a}{\partial \hat{k}} = 0.$$

Thus, when the shareholders choose an interior solution, an increase in the bank's equity funding costs reduces the level of actual capital, while an increase in the minimum capital requirement does not have any effect on their choice.

An important difference between economic and actual capital is that in choosing the former, bank shareholders have the option of providing no capital, which implies that the bank always has a positive franchise value, while in choosing the latter, they have to provide at least the minimum capital required by regulation, which in some cases may lead them to prefer not to operate the bank ($V^a = 0$). Whenever shareholders choose to operate the bank ($V^a > 0$), actual capital will, by construction, be greater than or equal to regulatory capital. In contrast, economic capital may be below regulatory capital. Obviously, the bank's franchise value will be, in general, higher for economic capital than for actual capital ($V^* > V^a$), because the constraints imposed by the regulator reduce the value of the bank.

It should also be noted that for $\hat{k}_{\min} = 0$, the bank's objective function in Bellman equation (15) coincides with the objective function in Bellman equation (10) that characterizes economic capital, except for the fact that in the former the bank's choice of capital is restricted to the interval $[\hat{k}, 1]$. But this implies that $k^a = \max\{k^*, \hat{k}\}$, except when $k^* < \hat{k}$ and the shareholders find it optimal not to operate the bank. In words, when the critical end-of-period capital $\hat{k}_{\min}$ below which the bank is closed by the supervisor is 0, and the shareholders choose to operate the bank, actual capital will be equal to the maximum of economic and regulatory capital.

Since comparative static results are, in general, ambiguous, in the following section we resort to numerical solutions to discuss the relationship between regulatory, economic, and actual capital.

3. Results

This section compares the values of regulatory capital $\hat{k}$, economic capital k^*, and actual capital k^a obtained by, respectively, computing

IRB formula (14), and solving Bellman equations (10) and (15), for plausible values of the parameters of the model.[9] The implicit assumption is that the bank invests all its portfolio in a single class of loans, with the same probability of default p and loss given default λ.

For the benchmark case, we assume a probability of default p of 2 percent and a loss given default λ of 45 percent (the value specified in the IRB foundation approach of Basel II for senior claims on corporates, sovereigns, and banks not secured by recognized collateral). For computing regulatory capital, we use the confidence level $\alpha = 99.9$ percent also set in Basel II.

The exposure-to-systematic-risk parameter ρ will be assumed to be a decreasing function of the probability of default p, according to the functional form specified in Basel II for corporate, sovereign, and bank exposures, which is

$$\rho(p) = 0.24 - 0.12\,\frac{1 - e^{-50p}}{1 - e^{-50}}.$$

Thus the maximum value of the exposure to systematic risk is $\rho(0) = 0.24$, the minimum value is $\rho(1) = 0.12$, and for the benchmark probability of default we have $\rho(0.02) = 0.16$. The effect of this assumption is to flatten (relative to the case with a constant ρ) the function that relates regulatory capital $\hat{k}$ to the probability of default p. However, our conclusions do not vary qualitatively when ρ is constant.

With regard to the loan rate r, instead of taking it as exogenous, we assume that it is determined by equating the expected return of a loan, $(1 - p)r - p\lambda$, to a margin μ over the risk-free rate, which is normalized to 0, i.e.,

$$(1 - p)r - p\lambda = \mu. \tag{16}$$

The margin μ is intended to capture the market power of the bank in the market for loans; i.e., it is taken to be exogenous.[10] Rearranging

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[9]Bellman equations (10) and (15) are solved by an iterative procedure. For example, in the case of (10), if we denote by $G(k, V)$ the bank's objective function, given an initial franchise value V_0, we compute $V_1 = \max_k G(k, V_0)$ and iterate the process until convergence to a value V^*. Economic capital is then given by $k^* = \arg\max_k G(k, V^*)$.

[10]Endogenizing μ would require an equilibrium model of imperfect competition in the loan market, which is beyond the scope of this paper.

the loan pricing equation, (16), we obtain

$$r = \frac{\mu + p\lambda}{1-p},$$

so the loan rate r is an increasing function of the probability of default p, the loss given default λ, and the intermediation margin μ. In the benchmark case we take a value of μ of 1 percent.

For the deposit rate c, we assume that the return required by depositors is equal to the risk-free rate, which has been normalized to 0, and we consider two alternative scenarios. In the first scenario, depositors are fully insured (at a 0 premium) by a deposit insurance agency, so the deposit rate c is equal to the risk-free rate, i.e., $c = 0$. In the second scenario, depositors are completely uninsured, so under the assumption of risk neutrality, the deposit rate c has to verify the participation constraint

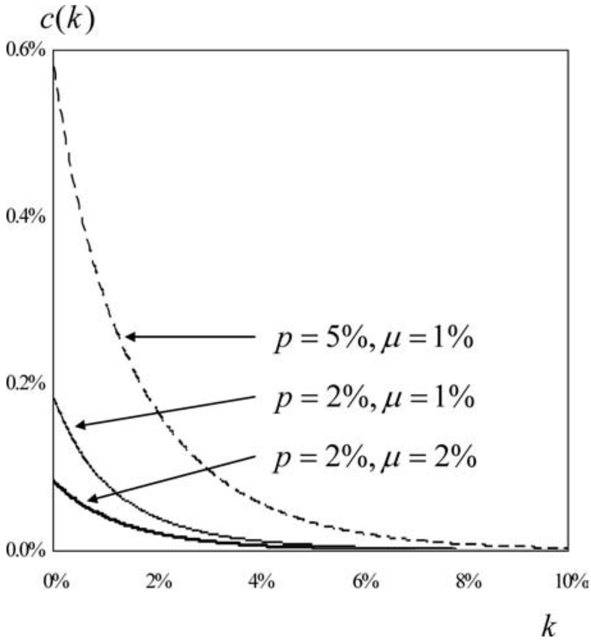
$$E[\min\{a, (1-k)(1+c)\}] = 1 - k. \tag{17}$$

To understand this equation, notice that when the value of the bank's end-of-period assets is greater than or equal to the depositors' principal and interest, i.e., when $k' = a - (1-k)(1+c) \geq 0$, depositors receive $(1-k)(1+c)$,[11] whereas when $k' < 0$, depositors receive the liquidation value of the bank, which (ignoring bankruptcy costs) is equal to a. Thus the left-hand side of equation (17) is the expected value of the depositors' claim at the end of each period, while the right-hand side is the gross return that they require on their investment. Appendix 2 shows that this equation has a unique solution $c(k) \geq 0$ for all k and that $c'(k) < 0$, except for $k \geq \lambda$, in which case $c'(k) = c(k) = 0$.

Figure 1 represents the cost of uninsured deposits c as a function of the capital level k—i.e., the function $c(k)$—for the benchmark case parameters, $p = 2$ percent and $\mu = 1$ percent, as well as the effects of an increase in the probability of default p and in the intermediation margin μ. The negative effect of k on the uninsured deposit rate c is significant for small values of k, for which the probability of bank failure is relatively high. An increase in the intermediation margin

[11] In the model of actual capital, the bank is closed for $0 \leq k' < \hat{k}_{\min}$, but in this case uninsured depositors also receive $(1-k)(1+c)$.

Figure 1. Effect of Bank Captial on the Uninsured Deposits' Interest Rate



μ from 1 percent to 2 percent reduces this probability and consequently the deposit rate c, whereas an increase in the probability of default p from 2 percent to 5 percent has the opposite effect.

The last parameter that has to be specified is the expected return δ required by bank shareholders, or cost of bank capital, which in the benchmark case is set equal to 6 percent.[12] Since we have normalized the risk-free rate to 0, this value should be interpreted as a spread over the risk-free rate.

Table 1 summarizes the parameter values in the benchmark case, as well as the range of values for which regulatory, economic, and

[12]Maccario, Sironi, and Zazzara (2002) estimate the cost of tier 1 capital for G-10 countries' major banks over the period 1993–2001, obtaining yearly averages between 6 and 10 percent. McCauley and Zimmer (1991) estimate banks' cost of equity for six countries during the period 1984–90, obtaining average estimates of around 10 percent for Canadian, UK, and U.S. banks; 6 percent for German and Swiss banks; and 3 percent for Japanese banks.

Table 1. Parameter Values Used in the Numerical Exercise

Parameter	Benchmark Case	Range of Values
Probability of Default p	2%	0–20%
Intermediation Margin μ	1%	0–5%
Cost of Bank Capital δ	6%	0–10%
Loss Given Default λ	45%	0–100%

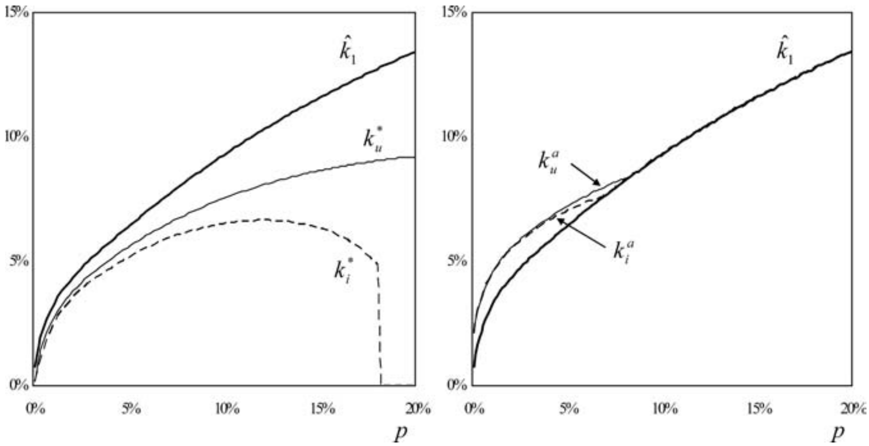
actual capital will be computed, keeping the rest of the parameters at their benchmark levels.

Our model only considers deposits and equity capital as sources of bank funding, but one should bear in mind that, in reality, there are many instruments in between. For regulatory purposes, Basel II distinguishes between tier 1 and tier 2 capital.[13] Tier 1 comprises equity capital and reserves from retained earnings, while tier 2 represents "supplementary capital" such as undisclosed reserves, revaluation reserves, general loan-loss reserves, hybrid (debt/equity) capital instruments, and subordinated debt. Ignoring for simplicity the special treatment of loan-loss provisions, Basel II involves two constraints: (i) tier 1 plus tier 2 capital should be greater than the minimum capital requirement, and (ii) tier 1 capital should be greater than 50 percent of the minimum requirement.

In what follows, we restrict attention to the tier 1 minimum capital requirement $\hat{k}_1 = \hat{k}/2$, where $\hat{k}$ is computed from IRB formula (14). In the case where deposits are uninsured, this requires no justification, since these deposits could be identified with subordinated debt, so tier 1 plus tier 2 capital would equal 100 percent of the bank's assets. In the case where deposits are insured, we would be effectively ignoring the tier 2 capital requirement. But since, as we will see below, the effect on the bank's capital choice of going

[13]The definition of eligible regulatory capital has not changed from Basel I; see Basel Committee on Banking Supervision (1988, paragraph 14; 2004, paragraphs 40 and 41).

Figure 2. Effect of the Loans' Probability of Default on Regulatory, Economic, and Actual Capital



from no insurance to full insurance is generally small, the effect of ignoring a small tier 2 requirement is negligible.

Finally, to compute actual capital, we follow FDICIA and set the threshold for critically undercapitalized banks at $\hat{k}_{\min} = 2$ percent.

3.1 Effect of the Loans' Probability of Default

The left panel of figure 2 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the loans' probability of default p, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of p.

As discussed in section 2, an increase in the probability of default p increases regulatory capital but has an ambiguous effect on economic capital. In particular, the left panel of figure 2 shows that economic capital with insured deposits k_i^* is increasing in the probability of default for values of p below 12 percent, is decreasing for values of p between 12 and 18 percent, and jumps to the corner solution $k_i^* = 0$ for higher values of p. Economic capital with uninsured deposits k_u^* is also first increasing and then decreasing in the probability of default p, although the change in slope takes place for much higher levels of p.

Economic capital with insured deposits k_i^* is always below economic capital with uninsured deposits k_u^*, because in the latter case shareholders have an additional incentive to provide capital in order to reduce the cost of uninsured deposits. This effect is more important when the loans' probability of default p is high because of the higher impact of the capital level k on the uninsured deposits' interest rate c noted above. Hence we conclude that the market discipline introduced by uninsured depositors leads to higher bank capital.

With respect to actual capital, the right panel of figure 2 shows that actual capital with insured and uninsured deposits, k_i^a and k_u^a, is strictly greater than regulatory capital $\hat{k}_1$ for default probabilities p below 7.7 and 8.5 percent, respectively. It can be shown that this buffer is increasing in the critical end-of-period capital $\hat{k}_{\min}$ below which the bank is closed by the supervisor, with $k_i^a = k_u^a = \hat{k}_1$ for $\hat{k}_{\min} = 0$. This result indicates that PCA provisions are an effective instrument to induce banks to hold capital levels above the minimum required by regulation.

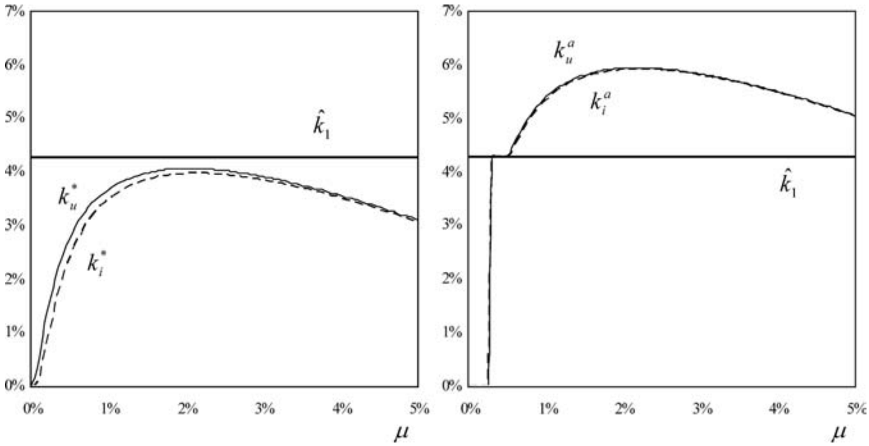
The right panel of figure 2 also shows that, for those cases where an interior solution exists, actual capital with insured deposits k_i^a is always below actual capital with uninsured deposits k_u^a. For higher default probabilities, those capital levels are equal to the minimum requirement $\hat{k}_1$, except when p is greater than 32 percent, in which case shareholders do not operate the bank when deposits are uninsured. Finally, the gap between actual capital with uninsured and insured deposits, $k_u^a - k_i^a$, is smaller than the corresponding gap for economic capital, $k_u^* - k_i^*$, because since actual capital is greater than economic capital, shareholders have fewer incentives to provide capital in order to reduce the cost of uninsured deposits. As we shall see below, this is a general result.

3.2 Effect of the Intermediation Margin

The left panel of figure 3 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the intermediation margin μ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of μ.

The intermediation margin μ has two opposite effects on economic capital. On the one hand, a higher margin increases the bank's

Figure 3. Effect of the Intermediation Margin on Regulatory, Economic, and Actual Capital



franchise value V and therefore shareholders' incentives to provide capital in order to preserve it. On the other hand, by assumption (16), a higher margin increases the loan rate r, which increases (in the sense of first-order stochastic dominance) the bank's portfolio return, and consequently reduces the need to hold capital in order to protect V.

The left panel of figure 3 shows that, for values of the intermediation margin μ below 2.1 percent, increases in the margin increase both k_i^* and k_u^*, bringing them closer to regulatory capital (which does not vary with μ), but the relationship becomes negative for higher values of the margin μ. Thus, for sufficiently competitive banking markets, the positive effect of the intermediation margin on economic capital (via an increase in the bank's franchise value) outweighs its negative effect (via the substitution between economic capital and the margin), while for oligopolistic markets, the negative effect dominates.

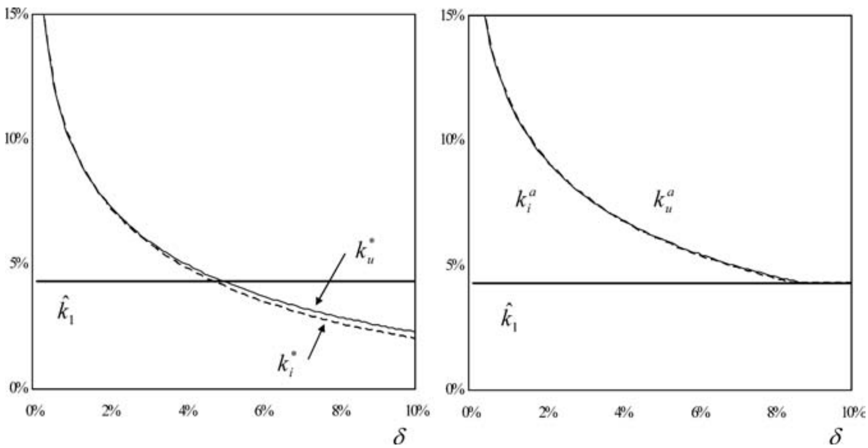
With respect to actual capital, the right panel of figure 3 shows that when the intermediation margin is below 0.25 percent, the shareholders prefer not to operate the bank rather than provide the minimum capital $\hat{k}_1$. Beyond this point, and for those values of the margin for which the restriction $k^a \geq \hat{k}_1$ is not binding, actual capital has a shape similar to that of economic capital. Again, whenever the bank operates, actual capital is higher than economic capital, which explains why the effect of the market discipline introduced by uninsured depositors on actual capital is almost negligible.

3.3 Effect of the Cost of Bank Capital

In all cases analyzed so far, we have found economic capital to be below regulatory capital. This is mainly due to our benchmark parameter value for the cost of bank capital δ. The left panel of figure 4 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the cost of capital δ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of δ.

As shown in appendix 1, economic capital is a decreasing function of the cost of capital ($\partial k^*/\partial\delta < 0$). Moreover, for values of the cost of capital δ below approximately 5 percent, both levels of economic capital, with and without insured deposits, are above regulatory capital. The reason is obvious: the lower the cost of capital δ, the higher the incentives of bank shareholders to contribute

Figure 4. Effect of the Cost of Bank Capital on Regulatory, Economic, and Actual Capital



capital. In fact, for values of δ sufficiently close to 0, shareholders choose capital levels that effectively guarantee the bank's survival regardless of the fraction of the loans in its portfolio that default.

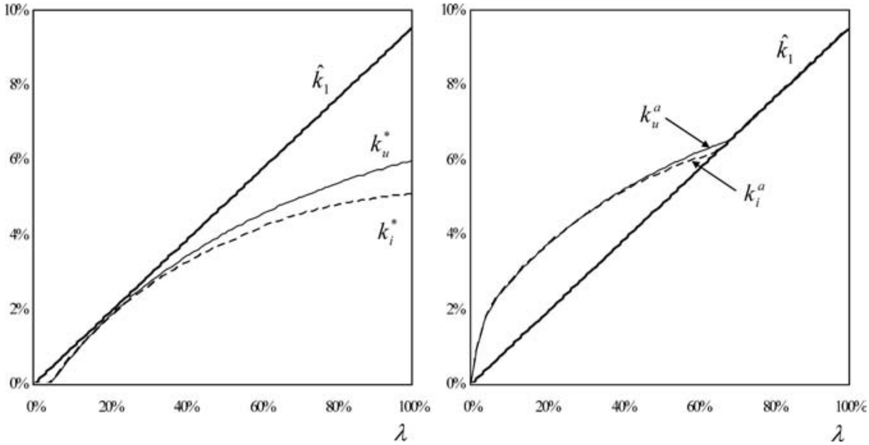
The relative position of actual capital with respect to economic and regulatory capital follows the same pattern as in figures 2 and 3. Actual capital is higher than regulatory capital for values of δ below 8.4 and 8.8 percent, respectively, for the insured and uninsured deposits cases. From those levels onward, they are equal to $\hat{k}_1$. The shareholders do not operate the bank for unreasonably high values of δ (above 23.5 percent).

3.4 Effect of the Loans' Loss Given Default

The left panel of figure 5 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the bank loans' loss given default λ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of λ.

According to IRB formula (14), regulatory capital $\hat{k}_1$ is a linear function of the loss given default λ. While the effect of λ on economic

Figure 5. Effect of the Loans' Loss Given Default on Regulatory, Economic, and Actual Capital



capital is positive in figure 5, as noted in section 2, this is not true in general (for example, if the probability of default p equals 7 percent, k_i^* starts to decrease for values of λ greater than 57 percent). Finally, actual capital is strictly above regulatory capital for most values of λ.

To sum up, we have found that both regulatory and economic capital depend positively on the loans' probability of default and loss given default for reasonable values of these variables. However, variables that only affect economic capital, such as the intermediation margin and the cost of capital, may significantly move it away from regulatory capital. Actual capital, which by definition is higher than regulatory capital, always lies above economic capital, and it is increasing in the critical capital level below which the supervisor closes the bank. We have also found that market discipline, proxied by the coverage of deposit insurance, has a positive impact on economic capital, but the effect is, in general, small and very sensitive to the values of the rest of the determinants of economic capital. Since actual capital is higher than economic capital, it is affected even less by market discipline.

4. Conclusion

Defining economic capital as the capital that shareholders would choose in the absence of regulation, this paper analyzes the determinants of economic and regulatory capital for a bank whose loan default rates are derived from the single-risk-factor model that underlies the capital charges in the IRB approach of Basel II. Our results show that there does not exist a direct relationship between both capital levels.

First, economic and regulatory capital do not depend on the same variables: regulatory (but not economic) capital depends on the confidence level set by the regulator, while economic (but not regulatory) capital depends on the intermediation margin and the cost of bank capital. These last two variables play a key role in determining the differences between economic and regulatory capital. Economic capital is above regulatory capital for low values of the cost of capital, and when this cost increases, the former quickly falls below the latter. The effect of the intermediation margin on economic capital is nonmonotonic, which is explained by the existence of two opposite effects: on the one hand, a higher margin increases the bank's franchise value and hence shareholders' incentives to contribute capital in order to preserve it, but on the other hand, a higher margin provides a source of income that reduces the need to hold capital as a buffer against losses. The first (positive) effect outweighs the second (negative) in sufficiently competitive credit markets. Therefore, changes in the market power of banks—due, e.g., to entry of new banks or mergers and acquisitions—may have very different effects on economic capital, depending on the initial level of competition.

Second, variables that affect both economic and regulatory capital, such as the loans' probability of default and loss given default, have a positive impact on both capital levels for reasonable values of these variables. But when they reach certain critical values, their effect on economic capital becomes negative, increasing the gap between economic and regulatory capital.

However, it is important to note that, in reality, banks choose their capital structure considering the regulations in place; i.e., they choose actual capital rather than economic capital. We define actual capital as the equity capital chosen by the bank shareholders when their choice is restricted by two regulations: (i) an initial capital greater than or equal to the minimum required by regulation and (ii) a closure rule for critically undercapitalized banks. The first regulation alone makes actual capital equal to the maximum of economic and regulatory capital, which according to our results coincides almost always with the latter (except for small values of the cost of capital). Therefore, whenever actual capital is higher than regulatory capital, this is likely to be explained by the second regulation. Our results indicate that the threat of closing critically undercapitalized banks (banks with tier 1 capital below 2 percent) significantly increases actual bank capital for reasonable ranges of parameter values. This regulation was introduced in the United States by the Federal Deposit Insurance Corporation Improvement Act, and it is not explicitly contemplated in Basel II. However, under pillar 2 (supervisory review process) of Basel II, national supervisors have discretion to introduce prompt corrective action provisions. According to our results, PCA provisions would be an effective instrument to induce banks to hold capital buffers.

Finally, the comparison of economic capital with insured and uninsured deposits reveals that, even though the latter is never below the former, their differences are, in general, small and very sensitive to the values of the rest of the determinants of economic capital. In the case of actual capital, those differences are almost negligible. Therefore, we conclude that the effects on the banks' capital structure of policies aimed at increasing market discipline, such as those contemplated in pillar 3 of Basel II, may be very limited.

Appendix 1. Comparative Statics of Economic Capital

This appendix discusses the effects on economic capital k^* of changes in its determinants—namely, the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ; the loan rate r; the deposit rate c; and the cost of bank capital δ. It is shown that when there is an interior solution ($k^* > 0$), we can only determine the effect of the cost of capital.

To this end we first note that, integrating by parts and taking into account the restriction $k \leq k_{\max}$, we can rewrite Bellman equation (12) as

$$V = \max_{k \in [0, k_{\max}]} G(k, V), \tag{18}$$

where

$$G(k, V) = -k + \frac{1}{1+\delta}\left[(\lambda + r)\int_0^{x(k)} F(x)dx + F(x(k))V\right]. \tag{19}$$

The derivatives of the function $G(k, V)$ with respect to k are given by

$$\frac{\partial G}{\partial k} = -1 + \frac{1+c}{1+\delta}\left[F(x(k)) + \frac{f(x(k))V}{\lambda + r}\right], \tag{20}$$

$$\frac{\partial^2 G}{\partial k^2} = \frac{(1+c)^2}{(1+\delta)(\lambda + r)}\left[f(x(k)) + \frac{f'(x(k))V}{\lambda + r}\right], \tag{21}$$

where $f(x) = F'(x)$ is the density function of the default rate and $f'(x)$ is its derivative. While the first term of (21) is non-negative (since $f(\cdot)$ is a density), the second term can be either positive or negative. Thus $G(k, V)$ is not, in general, a convex or a concave function of k, which implies that we may have both corner and interior solutions. However, since $F(x(k_{\max})) = F(1) = 1$ and $f(x(k_{\max})) = f(1) = 0$, our assumption $\delta > c$ implies that the derivative of $G(k, V)$ with respect to k evaluated at $k_{\max}$ is always negative, so a corner solution at $k_{\max}$ can be ruled out. Therefore, the only possible corner solution is $k^* = 0$.

If an interior solution exists, it would be characterized by the first-order condition $\partial G/\partial k = 0$ and the second-order condition $\partial^2 G/\partial k^2 < 0$. Differentiating the first-order condition gives

$$\frac{\partial k^*}{\partial z} = -\left(\frac{\partial^2 G}{\partial k^2}\right)^{-1} \left(\frac{\partial^2 G}{\partial k \partial z} + \frac{\partial^2 G}{\partial k \partial V}\frac{\partial V}{\partial z}\right),$$

where z is any of the six variables that determine economic capital k^*. Since

$$\frac{\partial^2 G}{\partial k \partial V} = \frac{(1+c)f(x(k))}{(1+\delta)(\lambda + r)} > 0,$$

and by the second-order condition we have $\partial^2 G/\partial k^2 < 0$, we need to find the signs of $\partial^2 G/\partial k \partial z$ and $\partial V/\partial \delta$. For $z = \delta$ it is easy to check that (20) implies

$$\frac{\partial^2 G}{\partial k \partial \delta} = -\frac{1+c}{(1+\delta)^2}\left[F(x(k)) + \frac{f(x(k))V}{\lambda + r}\right] = -\frac{1}{1+\delta} < 0,$$

and by the definition of the franchise value V, (18), and the envelope theorem, we have

$$\begin{aligned}\frac{\partial V}{\partial \delta} &= -\frac{1}{(1+\delta)^2}\left[1 - \frac{F(x(k))}{1+\delta}\right]^{-1} \\ &\quad \times \left[(\lambda + r)\int_0^{x(k)} F(x)\, dx + F(x(k))V\right] < 0,\end{aligned}$$

which implies $\partial k^*/\partial \delta < 0$. However, for $z = p$, λ, and ρ, the sign of $\partial^2 G/\partial k \partial z$ is ambiguous; for $z = r$ we have $\partial^2 G/\partial k \partial r < 0$ and $\partial V/\partial r > 0$; and for $z = c$ we have $\partial^2 G/\partial k \partial c > 0$ and $\partial V/\partial c < 0$, so we would need additional assumptions to get comparative statics results for these five variables.

Appendix 2. Uninsured Deposits' Interest Rate

The uninsured deposits' interest rate c is obtained by solving the participation constraint, (17), that equates the expected value of the depositors' claim at the end of each period, $E[\min\{a, (1-k)(1+c)\}]$, to the gross return that the depositors require on their investment, $1-k$. This appendix shows that the equation,

$$U(c,k) = E[\min\{a,(1-k)(1+c)\}] - (1-k) = 0, \tag{22}$$

has a unique solution $c(k) \geq 0$ for all k, and that $c'(k) < 0$, except for $k \geq \lambda$, in which case $c'(k) = c(k) = 0$.

For $k \geq \lambda$ it is immediate to check that $U(c,k) \geq U(0,k) = 0$, with strict inequality for $c > 0$, so $c = 0$ is the unique solution.

For $k < \lambda$, given that $0 \leq x(k) < 1$ for all $0 < c \leq (k+r)/(1-k)$, substituting the definition of a, (1), into (22), integrating by parts, and making use of the definition of $x(k)$, (11), gives

$$U(c,k) = k - \lambda + (\lambda + r)\int_{x(k)}^{1} F(x)\ dx. \tag{23}$$

To prove that (22) has a unique solution $c(k) > 0$, it suffices to show that $U(0,k) < 0 < \max_c U(c,k)$ and that $\partial U/\partial c > 0$. First, since $F(x(k)) < 1$, using (23) and the definition of $x(k)$, (11), implies

$$U(0,k) < k - \lambda + (\lambda + r)(1 - x(k)) = 0.$$

Second, using the fact $\int_0^1 F(x)\ dx = 1 - p$, together with the fact that by equation (16) we have $(1-p)r - p\lambda = \mu > 0$, (23) implies

$$\begin{aligned}\max_c U(c,k) &= k - \lambda + (\lambda + r)\int_0^1 F(x)\ dx = k + (1-p)r - p\lambda \\ &= k + \mu > 0.\end{aligned}$$

And third, differentiating (23) with respect to c gives

$$\frac{\partial U}{\partial c} = (1-k)F(p(k)) > 0. \tag{24}$$

Finally, totally differentiating $U(c,k) = 0$ and using (24), we have $c'(k) < 0$ if

$$\frac{\partial U}{\partial k} = 1 - (1+c)F(x(k)) > 0.$$

But for $c = c(k)$, we have

$$U(c,k) = (1-k)(1+c)F(x(k)) + \int_{x(k)}^{1} a\ dF(x) - (1-k) = 0,$$

which implies

$$1 - (1+c)F(x(k)) = \frac{1}{1-k}\int_{x(k)}^{1} a\ dF(x) > 0.$$

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